Rectilinear motion:
$x(t)$ or $s(t)$ is the position of a particle at time t.

$x'(t) = v(t)$ is the velocity of the particle at time t..

$x''(t) = v'(t) = a(t)$ is the acceleration of the particle at time t.

Therefore $v(t) = \int a(t)\,dt \text{ and } x(t) = \int v(t)\,dt$

Displacement of a particle on the time interval $[a,b]$ is its change in position over the interval. For a particle moving horizontally, the displacement is positive if the final position is to the right of the initial position. It is negative if it ends up to the left and zero if they are the same.

$$\text{Displacement } = x(b) - x(a) \text{ or Displacement} = \int_a^b v(t)\,dt$$

Distance traveled by a particle is the sum of the distances traveled between endpoints and critical points ($v(t) = 0 \text{ or undefined}$. Example if $v(t) = 0 \text{ or undefined}$ at $t = c \text{ and } d$ then

$$\text{Distance } = |x(a) - x(c)| + |x(c) - x(d)| + |x(d) - x(b)| \text{ or Distance} = \int_a^b |v(t)|\,dt$$

$$\text{Average velocity} = \frac{x(b) - x(a)}{b - a} \text{ or } \frac{1}{b-a}\int_a^b v(t)\,dt$$

$$\text{Average acceleration} = \frac{v(b) - v(a)}{b - a} \text{ or } \frac{1}{b-a}\int_a^b a(t)\,dt$$

Initial condition formula $s(t) = s(t_0) + \int_{t_0}^{t} v(x)\,dx$

Or for velocity $v(t) = v(t_0) + \int_{t_0}^{t} a(x)\,dx$

Rectilinear motion:

Analyze the functions: Find 1st and 2nd derivatives, critical points and points of inflection, increasing and decreasing, concave up or down, relative max/min, and absolute max/min. Make a rough sketch of the function on its domain.

1) $f(x) = 2x^3 - 21x^2 + 60x + 3;\ [0,6]$

Consider the same function as particle motion. Now let us plot it position with respect to time, when the velocity is zero, positive, and negative, when the acceleration is zero, positive, and negative, distance traveled, displacement, and when the particle is speeding up or down.

2) $x(t) = 2t^3 - 21t^2 + 60t + 3;\ [0,6]$

Find its position with respect to time, when the velocity is zero, positive, and negative, when the acceleration is zero, positive, and negative, distance traveled, displacement, and when the particle is speeding up or down.

3) $s(t) = t^3 - 6t^2, [0, 7]$

Find its position with respect to time, when the velocity is zero, positive, and negative, when the acceleration is zero, positive, and negative, distance traveled, displacement, and when the particle is speeding up or down.

4) $s(t) = 3\cos\left(\frac{\pi t}{2}\right), \;\; 0 \le t \le 5$

Recognizing Derivatives & Integrals ________________

Differentiate the following:

1) $f(x) = e^{2\ln(3x+1)}$

2) $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x$

3) $y = \dfrac{5}{x^2} - \ln\left(\dfrac{\cos x}{\sin x + x}\right)$

4) $y = \dfrac{e^x + 9}{e^{x^2} - x^4}$

5) $f(x) = \dfrac{3x^2 - 5}{7}$

6) $f(x) = 3x\left(6x - 5x^2\right)$

7) $f(x) = \left(\dfrac{x+5}{x^2 - 7}\right)^2$

8) $f(x) = \dfrac{x}{\sqrt{x^4 - 5}}$

9) $f(x) = 7\tan^2(3x)$

10) $y = \ln\left(2x^2 + 1\right)$

11) $f(x) = \dfrac{\sec x}{x}$

12) $\ln y + xy^2 - 4x^3 + 10 = 3x$

13) $y = \left(x^2 + 6\right)\ln 3x$

14) $f(x) = (9x - 4)^{\frac{2}{3}}$

15) $f(x) = 2x\sin x + x^2\cos 3x$

16) $y = \dfrac{\cos x}{\sin x}$

17) $y = x^{\tan x}$

18) $f(x) = \cos x(\tan x - \sec x)$

Integration:

1) $\int \frac{x^2+2x-3}{x^4}\,dx$

2) $\int e^{\sec 2x}\sec 2x\tan 2x\,dx$

3) $\int \sec y(\tan y-\sec y)\,dy$

4) $\int e^{3x}dx$

5) $\int (\tan^2\alpha+1)\,d\alpha$

6) $\int t^3\sqrt{t^4+2}\ dt$

7) $\int \frac{(\ln x)^2}{x}\,dx$

8) $\int e^{-x}\tan\left(e^{-x}\right)dx$

9) $\int \frac{x}{\sqrt{2x-1}}\,dx$

10) $\int \frac{1}{3x+2}\,dx$

11) $\int \cos 6\vartheta\ d\vartheta$

12) $\int \cot x\,dx$

13) $\int \frac{1}{x^{\frac{2}{3}}\left(1+x^{\frac{1}{3}}\right)}\,dx$

14) $\int \sqrt[3]{x^2}\ dx$

15) $\int \frac{e^{2y}}{1-e^{2y}}\,dy$

16) $\int \frac{\csc^2 x}{\cot^3 x}\,dx$

17) $\int ((x+1)(2x-3))dx$

18) $\int \frac{e^{3x}-2e^x+5}{e^{2x}}\,dx$